The question of the variation of (1) the direct and (2) the diffused radiation is next discussed. On comparing the curve representing the chemical intensity of diffused light with the curve of solar obscuration, it is found that the rate of diminution in chemical action exerted by the diffused light is up to a certain point greater than corresponds to the portion of eclipsed sun, whilst from this point up to totality the rate of diminution becomes less than corresponds to the progress of the eclipse. The same rapid diminution in the chemical action of the diffused daylight during the early periods of the eclipse was also observed at Jamkandi; it is doubtless due to the dark body of the moon cutting off the light from the brightly illuminated portion of sky lying round the solar disk.

The results of the observations at Catania are then compared with those made at Moita, near Lisbon, and communicated to the Society in 1870. This comparison shows a striking coincidence between the two sets of observations. In each case it is seen that the relation between solar altitude and total chemical intensity is represented by a straight line, although the Catania observations slightly exceed, by a constant difference, those made at Moita in conformity with the slight difference in latitude, and with the fact that the former determinations were made at a greater elevation above the sea-level.

The Catania observations further confirm the fact which we formerly announced, that for altitudes below 50° the amount of chemical action effected in the plane of the horizon by diffused daylight is greater than that exerted by direct radiation, and also that at altitudes below 10° direct sunlight is almost completely robbed of its chemically active rays.

VII. "On the Calculation of Euler's Constant." By J. W. L. GLAISHER, B.A., F.R.A.S. Communicated by JAMES GLAISHER, F.R.S. Received June 6, 1871.

The main object of the present communication is to correct some inaccuracies both of reasoning and calculation contained in two papers by Mr. Shanks, viz. "On the Extension of the Value of the Base of Napier's Logarithms; of the Napierian Logarithms of 2, 3, 5, and 10; and of the Modulus of Briggs on the common System of Logarithms; all to 205 places of Decimals," in the Proc. Roy. Soc. vol. vi. p. 397; and "On the Calculation of the Numerical Value of Euler's Constant," in the Proc. Roy. Soc. vol. xv. p. 429 (1867).

For the calculation of the constant Mr. Shanks has used (as, indeed, has every calculator who has computed the value of the constant during the present century) the semiconvergent series

$$\gamma=1+\frac{1}{2}+\frac{1}{3}\ldots+\frac{1}{x}-\log x-\frac{1}{2x}+\frac{B_1}{2x^2}-\frac{B_2}{4x^4}+\frac{B_3}{6x^6}-\ldots, \qquad \text{(i)}$$

γ being the constant, and B_1, B_2, B_3 . . . Bernoulli's numbers.

Mr. Shanks obtains the value of γ from this formula by making $x=10$, 20, 50, 100, 200, 500, and 1000, and remarks, as a curious coincidence, that the number of decimal places obtained from x being made equal to 10, 20, 50, and 100 is nearly proportional to $\sqrt[3]{10}$, $\sqrt[3]{20}$, $\sqrt[3]{50}$, and $\sqrt[3]{100}$. On p. 432 he gives a Table of the number of decimal places obtainable from the formula when x has the values 2, 5, 10, 20, 50, 100, 200, 500, and 1000, and from it draws the inference that "we may fairly infer that when n is increased in a geometrical ratio, the corresponding number of decimals obtained in the value of E increases only in something like an arithmetical one, and that probably from 50,000 to 100,000 terms in the Harmonic Progression would require to be summed in order to obtain 100 places of decimals in the value of E, Euler's constant."

Algebraically, of course, γ is independent of x in the formula (i), but arithmetically, since the series ultimately becomes divergent, the value of γ is so far dependent on x that for a given value of x the series will only afford a certain number of decimal places. The number of decimal places directly obtainable is equal to the number of ciphers which precede the first significant figure in the value of the numerically least term of the series.

The nth term (considering only the terms after $\frac{1}{2x}$ in (i), so that the nth term is the term involving B_n)

$$=\frac{B_n}{2nx^{2n}},$$

which, since

$$B_n=\frac{2(1.2.3\ldots 2n)}{(2\pi)^{2n}}\left(1+\frac{1}{2^{2n}}+\frac{1}{3^{2n}}+\ldots\right),$$

is very nearly equal to

$$\frac{2\{1.2.3\ldots(2n-1)\}}{(2\pi x)^{2n}};$$

so that the ratio of the nth to the $(n-1)$th term

$$=\frac{(2n-2)(2n-1)}{4\pi^2x^2}=\frac{n^2}{\pi^2x^2}\left(1-\frac{3}{2n}\right)$$

very nearly.

Let m be the greatest integer contained in $x\pi$ so that $x\pi=m+f$, f being a proper fraction, then the ratio of the mth to the $(m-1)$th term

$$=\frac{m^2}{(m+f)^2}\left(1-\frac{3}{2m}\right)$$

$$=1-\frac{4f+3}{2m},$$

which is always less than unity.

The ratio of the $(m+1)$th to the mth term is found in a similar manner to be

$$1+\frac{2}{m}\left(\frac{1}{4}-f\right),$$

which is greater or less than unity according as f is less or greater than $\frac{1}{4}$.

The ratio of the $(m+2)$th to the $(m+1)$th term

$$=1+\frac{2}{m}\left(\frac{5}{4}-f\right),$$

which is always greater than unity.

Thus the mth or $(m+1)$th term is the least according as f is less or greater than $\frac{1}{4}$.

When m is large the value of the mth term is very nearly

$$\frac{1}{m}\cdot\frac{1\,.\,2\,.\,3\,.\,.\,.\,.\,2m}{(2\pi x)^{2m}},$$

which, by use of the theorem

$$1\,.\,2\,.\,3\,.\,.\,.\,x=\sqrt{(2\pi x)}\,x^{x}e^{-x}\left(1+\frac{1}{12x}\right),$$

becomes

$$\frac{1}{\sqrt{m}}\cdot\frac{2\sqrt{\pi}(2m)^{2m}e^{-2m}}{(2\pi x)^{2m}}\left(1+\frac{1}{24m}\right)$$

$$=2\sqrt{\frac{\pi}{m}}\cdot\left(\frac{m}{\pi e x}\right)^{2m}\left(1+\frac{1}{24m}\right).\quad .\ .\ .\ .\ .\ \text{(ii)}$$

The number which forms the negative characteristic of the common logarithm of this quantity (the mantissa being made positive), reduced by unity, denotes the number of ciphers which precede the first significant figure in its value; so that if the mantissa be not made positive, the characteristic indicates the number of decimals directly obtainable from the series.

The logarithm of (ii) with its sign changed, after some expansions and reductions, becomes

$$2\mu\pi x+\frac{1}{2}\log_{10}x-\log_{10}2-\mu\left(\frac{f}{2x\pi}+\frac{f^{2}}{x\pi}+\frac{1}{24x\pi}\right),$$

μ being the modulus 43429448 . . . . Neglecting the last three terms, which can only very rarely lead to an error of a unit, we obtain as a result that the number of decimal places which the formula (i) will afford directly for a given value of x is equal to the greatest integer contained in

$$2\mu\pi x+\frac{1}{2}\log_{10}x-\log_{10}2.\quad .\ .\ .\ .\ .\ .\ .\ \text{(iii)}$$

The corresponding value of the least term is, of course, easily obtained from (iii), or it follows at once from (ii); for the least term

$$=2\sqrt{\left(\frac{\pi}{m}\right)}e^{-2m}\left(\frac{m}{m+f}\right)^{2m}$$

when m is large,

$$=2\sqrt{\left(\frac{\pi}{m}\right)}e^{-2m}\left(\frac{1}{1+\frac{f}{m}}\right)^{2m},$$

$$=2\sqrt{\left(\frac{\pi}{m}\right)}e^{-2m-2f}$$

$$=\frac{2}{\sqrt{x}}e^{-2\pi x},$$

agreeing with (iii).

The following Table, to replace that in vol. xv. p. 432, was calculated in the way indicated above.

x.		Least term.		Number of decimals directly obtainable from the formula.
1	......	3rd	......	2
2	......	7th	......	5
5	......	16th	......	13
10	......	32nd	......	27
20	......	63rd	......	54
50	......	157th	......	136
100	......	314th	......	273
200	......	629th	......	546
500	......	1571st	......	1365
1000	......	3142nd	......	2729

The number of decimal places practically obtainable is limited by the difficulty of calculating the Bernoulli's numbers, of which only thirty-one have been hitherto obtained. By means of these, however, 156 decimals could be obtained of γ when $x=100$; and by deducing a few of the subsequent terms, each from its predecessor (knowing their ratio), 20 places more could be obtained without difficulty.

It is clear therefore that Mr. Shanks's values of γ obtained from $x=500$ and $x=1000$ ought to agree beyond the 59th decimal, if correctly calculated. The author at first supposed that the want of agreement was due to an insufficient number of the terms involving the Bernoulli's numbers having been included, and he therefore undertook the calculation of this portion of the expression for $x=500$ and $x=1000$ to 100 decimal places; the results, however, still showed a difference in the 59th place.

In order to determine where the error existed, the same portion of the constant was calculated also to 100 places, both from $x=100$ and $x=200$; all the calculations were performed wholly in duplicate, and so much care was taken that the author felt a strong conviction of their accuracy. It should be noticed that the agreement of all the four results would not necessarily prove the accuracy of that value of γ; for any error made in the calculation of the harmonic series $1+\frac{1}{2}\ldots\ldots+\frac{1}{100}$ would merely produce the same error in $1+\frac{1}{2}\ldots+\frac{1}{200}$, $1+\frac{1}{2}\ldots+\frac{1}{500}$, and $1+\frac{1}{2}\ldots+\frac{1}{1000}$, supposing the calculation of the reciprocals beyond $\frac{1}{100}$ to have been accurately performed.

It was therefore evident that the only means of ensuring freedom from error in the harmonic series was to recalculate it. The lowest value of x which would suffice for a verification was 100; the author therefore calculated the value of the series $1+\frac{1}{2}\ldots+\frac{1}{100}$ to 100 places of decimals, and the result was found to agree to that extent with that given by Mr. Shanks in the paper previously referred to; the value of $1+\frac{1}{2}\ldots+\frac{1}{50}$ was also found to be correct.

It may be mentioned that the calculation was abbreviated by a simple artifice, suggested by Oettinger (Crelle's Journal, t. lx. p. 376), which will be easily understood from an example. Suppose the sums of the reciprocals of the odd and even numbers up to 50 are known, and it is required to find the sum of the reciprocals up to 100.

Let

$$\alpha=1+\frac{1}{3}+\frac{1}{5}\ldots\frac{1}{49},$$

$$\beta=\frac{1}{2}+\frac{1}{4}+\frac{1}{6}\ldots\frac{1}{50};$$

then

$$\frac{\alpha+\beta}{2}=\frac{1}{2}+\frac{1}{4}+\frac{1}{6}\ldots+\frac{1}{98}+\frac{1}{100};$$

so that

$$1+\frac{1}{2}+\frac{1}{3}\ldots+\frac{1}{99}+\frac{1}{100}$$
$$=\frac{\alpha+\beta}{2}+\alpha+\frac{1}{51}+\frac{1}{53}\ldots+\frac{1}{99},$$

and the calculation of the reciprocals of the even numbers between 50 and 100 is rendered unnecessary.

The values of log 2 and log 5, which were required to form log 100, log 200, log 500, and log 1000 were taken from Mr. Shanks's 'Rectification of the Circle' (they are also given in the Proc. Roy. Soc. vol. vi. p. 397), and the summations of the harmonic series from Mr. Shanks's paper in vol. xv. p. 431 of the Proc. Roy. Soc. The results were as follows:—

From $x = 100$:—

$$\begin{array}{lllllll}\gamma = & \cdot 57721 & 56649 & 01532 & 86060 & 65120 & \\ & 90082 & 40243 & 10421 & 59335 & 93992 & \\ & 35988 & 05770 & 42799 & 90853 & 75858 & \\ & 22362 & 13134 & 84454 & 84292 & 91195 & \ldots \text{(A).}\end{array}$$

From $x = 200$:—

$\gamma = \cdot 57721$ 56649 01532 86060 65120
90082 40243 10421 59335 93995
35988 05770 91390 77706 70283
52607 02838 27101 38332 64367 . . . (B).

From $x = 500$:—

$\gamma = \cdot 57721$ 56649 01532 86060 65120
90082 40243 10421 59335 93995
35988 05771 53864 75089 05973
20064 75314 10504 07812 29873 . . . (C).

From $x = 1000$:—

$\gamma = \cdot 57721$ 56649 01532 86060 65120
90082 40243 10421 59335 93995
35988 05772 02455 61942 00398
50309 65017 53150 61852 03044 . . . (D).

The terms involving B_{39}, B_{31}, B_{22}, and B_{19} were the highest used in the calculation of (A), (B), (C), and (D) respectively.

It will be seen that (A), (B), (C), and (D) differ in two respects: (i) the 50th figure in (A) is 2, while in (B), (C), and (D) it is 5, and (ii) all four values are totally different after the 59th figure.

The first discrepancy (i) pointed to an error in the summation of the harmonic series. As the author had verified Mr. Shanks's value of $1+\frac{1}{2}\ldots+\frac{1}{100}$, it was practically certain that (A) was the correct value. To place this beyond all doubt, however, the author calculated γ from $x=50$ to 57 places of decimals, and the result entirely confirmed (A). It follows, therefore, that Mr. Shanks has made an error of 3 in the 50th place in the calculation $\frac{1}{100}+\frac{1}{101}\ldots+\frac{1}{200}$, so that we have the following errata (vol. xv. p. 431) :—In $1+\frac{1}{2}\ldots+\frac{1}{200}$, the tenth group of five should be . . . 53024 . . . instead of . . . 53027 . . .

In $1+\frac{1}{2}\ldots+\frac{1}{500}$ the tenth group should be . . . 39677 . . . instead of . . . 39680 . . .

In $1+\frac{1}{2}\ldots+\frac{1}{1000}$ the tenth group should be . . . 70880 . . . instead of . . . 70883 . . .

With regard to the second error (ii), which causes the disagreement of all the figures after the 59th, it is clear that no error in the Bernoulli's numbers could produce such a discrepancy, as the terms involving the latter vary their position in each calculation, so that an error in any one of them could not affect the same decimal in (A), (B), (C), and (D).

By subtraction we find :

$$\begin{array}{llllll} (B)-(A)= \ldots \text{ (60 ciphers) .} & & 48590 & 86852 & & \\ & 94425 & 30244 & 89703 & 42646 & \\ & 54039 & 73172 & \ldots\ldots\ldots & & (E) \\ (D)-(C)= \ldots \text{ (60 ciphers)} \ldots & & 48590 & 86852 & & \\ & 94425 & 30244 & 89703 & 42646 & \\ & 54039 & 73171 & \ldots\ldots\ldots & & (E) \end{array}$$

which only differ by a unit in the 100th decimal place.

Leaving out of consideration the terms involving Bernoulli's numbers, the difference between the series when $x=200$ and when $x=100$ is

$$\left(1+\frac{1}{2}\cdots+\frac{1}{200}-\log 200\right)-\left(1+\frac{1}{2}\cdots+\frac{1}{100}-\log 100\right)$$

$$=\frac{1}{101}+\frac{1}{102}\cdots+\frac{1}{200}-\log 2.$$

Similarly, the difference between the series when $x=1000$ and when $x=500$ is

$$=\frac{1}{501}+\frac{1}{502}\cdots+\frac{1}{1000}-\log 2.$$

Now as it is extremely improbable that two errors, exactly equal in amount, should have been made in the calculations of

$$\frac{1}{101}+\frac{1}{102}\cdots+\frac{1}{200} \text{ and } \frac{1}{501}+\frac{1}{502}\cdots+\frac{1}{1000},$$

we have very strong evidence that the value of log 2 is inaccurate, and that (E) is the correction to be applied to it.

By subtracting (C) from (A), we obtain

$$\begin{array}{llllll} (C)-(A)= \ldots \text{ (59 ciphers)} \ldots 1 & & 11064 & 84235 & & \\ 30114 & 97702 & 62179 & 26049 & 23519 & \\ 38678 & \ldots\ldots\ldots & & & & (F) \end{array}$$

The difference of the corresponding series (omitting as before the terms involving the Bernoulli's numbers)

$$=\frac{1}{101}+\frac{1}{102}\cdots+\frac{1}{500}-\log 5.$$

Having only this one* difference-result involving log 5, it is impossible to decide from (A), (B), (C), and (D) whether the harmonic series or log 5 or both are in error ; but the following reasoning places it beyond all doubt that (F) is a correction to log 5, and that the sum of the harmonic series is correct.

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* By subtracting (B) from (D) we might get another, but the portion $\frac{1}{201}+\frac{1}{202}+\frac{1}{500}$ of the harmonic series, as well as log 5, would be common to both.—June 16.

Mr. Shanks computed log 2 and log 5 from formulæ of the form

$$\text{Log } 2=2\ (7\ P+5\ Q+3\ R),$$
$$\text{Log } 5=2\ (16\ P+12\ Q+7\ R),$$

P, Q, and R being infinite series (Proc. Roy. Soc. vol. vi. p. 397; 'Rectification of the Circle,' p. 88).

If, therefore, any error was made in the calculation of P say, it would produce errors in log 2 and log 5 proportional to 7 and 16. On trial it was found that sixteen times (E) was equal to seven times (F), the difference being only such as an error of a unit in the 100th decimal of (E) or (F) would produce. This afforded a moral proof that Mr. Shanks had made an error equal to one-fourteenth of (E) in the calculation of P, or

$$\frac{1}{31}+\frac{1}{3.31^3}+\frac{1}{5.31^5}+\cdots$$

which has rendered his values of log 2 and log 5 incorrect, and that (with the exception of the error previously noticed) the harmonic series was summed correctly.

Since log 3 was calculated from the formula

$$\text{Log } 3=2\ (11\ P+8\ Q+5\ R),$$

its value is also incorrect, as also is that of log 10 (log 2 + log 5) and the modulus (the reciprocal of log 10).

The values of all these quantities are given to 205 decimal places in vol. vi. Proceedings of the Royal Society, p. 397; but all the figures after the 59th decimal place are incorrect in each case.

The correct values to 100 places are :—

Log 2=·69314	71805	59945	30941	72321
21458	17656	80755	00134	36025
52541	20680	00949	33936	21969
69471	56058	63326	99641	86875 . . .
Log 3=1·09861	22886	68109	69139	52452
36922	52570	46474	90557	82274
94517	34694	33363	74942	93218
60896	68736	15754	81373	20888 . . .
Log 5=1·60943	79124	34100	37460	07593
33226	18763	95256	01354	26851
77219	12647	89147	41789	87707
65776	46301	33878	09317	96108 . . .
Log 10=2·30258	50929	94045	68401	79914
54684	36420	76011	01488	62877
29760	33327	90096	75726	09677
35248	02359	97205	08959	82982 . . .

It is to be observed that the above value of log 3 is not quite as well determined as the others; the calculations in regard to Euler's constant form a real verification of log 2, log 5, and log 10; they also verify P, Q, and R; but an error in log 3 in the transcription of P, Q, and R, or their multiplication by 11, 8, and 5, or in the final addition and multiplication by 2, would not be detected.

In the above logarithms the last figure may be in error to the extent of one or two units.

The value of Euler's constant to 100 decimal places is—

$$\begin{array}{llllll}\gamma = \cdot 57721 & 56649 & 01532 & 86060 & 65120 \\ 90082 & 40243 & 10421 & 59335 & 93992 \\ 35988 & 05767 & 23488 & 48677 & 26777 \\ 66467 & 09369 & 47063 & 29174 & 67497\ldots\end{array}$$

The last figure here also may be in error to the extent of one or two units.

It will be observed that Mr. Shanks's value* of γ for $x=500$ differs from (C) in the 65th decimal place, and that his value for $x=1000$ differs from (D) in the 73rd place. This is caused by an inaccurate value of B_{13} having been made use of. The correct value of B_{13} is $\frac{8553103}{6}$; but Euler, who first calculated it, made it $\frac{8553103}{2}$ ('Acta Petropolitana' for 1781, p. 46), and this incorrect value is given in the 'Penny Cyclopædia' (Article "Numbers of Bernoulli") and probably in other places.

The values of the first thirty-one Bernoulli's numbers are given in a paper by Ohm (Crelle's Journal, t. xx. p. 11), and B_{13} is given correctly there. The agreement of the values of Euler's constant contained in this paper (when the logarithms of 2 and 5 are corrected) afford a complete verification of the Bernoulli's numbers as far as B_{26}, and partial verifications of the rest.

The difficulty and inconvenience of making calculations to so many decimal places is sufficient to warrant the publication of the values of the positive and negative parts of the portion of the series involving the Bernoulli's numbers, in case any one should desire to repeat any part of the calculation. We have

$$\gamma = 1 + \frac{1}{2} + \frac{1}{3} \cdots + \frac{1}{x} - \log x - \frac{1}{2x} + \frac{B_1}{2x^2} - \frac{B_2}{4x^4} + \frac{B_3}{6x^6} - \cdots;$$

and if m denote the sum of the terms of the same sign as the harmonic series, and n the sum of the terms of the same sign as the logarithms, viz. if

$$m = \frac{B_1}{2x_2} + \frac{B_3}{6x^6} + \frac{B_5}{10x^{10}} + \cdots$$

$$n = \frac{1}{2x} + \frac{B_2}{4x^4} + \frac{B_4}{8x_8} + \cdots;$$

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* Proc. Roy. Soc. vol. xv. p. 432.

then, when $x=100$:—

$m=$·00000 83333 33337 30158 73773
44885 67821 37321 67823 08773
00082 30639 33761 49846 36254
14224 82920 15089 52013 00966
589 . . .

$n=$·00500 00000 83333 33375 00000
21092 80052 53967 77636 59871
21105 34750 25738 21912 47654
32311 34887 61384 97230 71640
133 . . .

When $x=200$:—

$m=$·00000 20833 33333 39533 73016
61283 59538 59711 91078 12258
22890 92137 92329 17052 51095
23490 89894 78957 35141 06774
044 . . .

$n=$·00250 00000 05208 33333 49609
37505 14960 84887 28877 09510
05685 64006 82170 11770 13023
01074 35246 38458 75084 75675
042 . . .

When $x=500$:—

$m=$·00000 03333 33333 33358 73015
87309 34487 73462 42678 21147
87864 83345 53789 61875 16264
59799 20761 22352 44796 75809
196 . . .

$n=$·00100 00000 00133 33333 33344
00000 00008 63960 92825 14227
06303 41363 50645 63522 06215
12108 84946 96404 61971 02278
386 . . .

When $x=1000$:—

$m=$·00000 00833 33333 33333 73015
87301 59487 73448 77428 21067
82137 32165 00876 22580 99464
52882 37182 41954 63931 48991
922 . . .

$n=$·00050 00000 00008 33333 33333
37500 00000 00210 92796 09323
93526 00040 82093 23718 92820
00915 30332 37713 49061 83360
485 . . .

The calculation was performed to 105 places, and the last two figures have been rejected.

POSTSCRIPT.

Received June 14, 1871.

After the completion of the above paper, the author found that Mr. Shanks had, in second, third, and fourth supplementary papers on the Constant*, extended his calculations so as to determine γ from $x=2000$, 5000, and 10,000.

The values so obtained all differ in the sixtieth decimal; in fact the higher x is taken, the further from the truth are the results, as the errors in the logarithms are multiplied by larger factors.

The calculation for $x=2000$ affords a verification of the error of log 2; for on subtracting the value of γ $(x=1000)$ from γ $(x=2000)$, we obtain (after correcting B_{13}) a result agreeing with E to 80 decimal places (which is as far as Mr. Shanks has calculated the latter value of γ), with the exception of a difference of a unit in the 73rd figure—an error probably in the summation of the harmonic series from 1000 to 2000.

The values for $x=5000$ and $x=10{,}000$ are (besides the errors previously noticed) inaccurate from the 62nd figure.

VIII. "Records of the Magnetic Observations at the Kew Observatory. No. IV.—Analysis of the principal Disturbances shown by the Horizontal and Vertical Force Magnetometers of the Kew Observatory from 1859 to 1864." By General Sir EDWARD SABINE, K.C.B., President. Received June 15, 1871.

(Abstract.)

This paper exhibits an analysis of the principal disturbances recorded by the horizontal and vertical force self-recording magnetometers of the Kew Observatory in the years 1859 to 1864, showing the progressive diminution in the number and value of the disturbances from a maximum in 1859 to a minimum in 1863, being the first moiety of the "decennial period;" and showing also the distribution of the disturbances, increasing or diminishing the respective forces, in the several years, months, and hours.

IX. "Amended Rule for working out Sumner's Method of finding a Ship's Place." By Prof. Sir WILLIAM THOMSON, F.R.S. Received June 15, 1871.

In my previous communication on this subject (*anteà*, p. 259) I described a plan according to which, in the first place, two auxiliary lines were to be drawn on the chart, from two sets of numbers taken out of a proposed Table, and then Sumner's line (the line on which the observation shows the ship to be) was to be interpolated, dividing the space between them in the proportion of the differences of the sun's decli-

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* Proc. Roy. Soc. vol. xvi. pp. 154, 299, vol. xviii. p. 49.